积分表

维基百科，自由的百科全书

由于列表比较长，**积分表**被分为以下几个部分：

- 有理函数积分表
- 无理函数积分表
- 三角函数积分表
- 指数函数积分表
- 对数函数积分表
- 反三角函数积分表
- 双曲函数积分表
- 反双曲函数积分表

目录

含有$ax+b$的积分

$$\int (ax+b)^n \mathrm{d}x = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int \frac{1}{ax+b}\mathrm{d}x = \frac{1}{a}\ln|ax+b| + C$$

$$\int \frac{x}{ax+b}\mathrm{d}x = \frac{1}{a^2}(ax+b-b\ln|ax+b|) + C$$

$$\int \frac{x^2}{ax+b}\mathrm{d}x = \frac{1}{2a^3}\left[(ax+b)^2 - 4b(ax+b) + 2b^2\ln|ax+b|\right] + C$$

$$\int \frac{1}{x(ax+b)}\mathrm{d}x = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right| + C$$

$$\int \frac{1}{x^2(ax+b)}\mathrm{d}x = \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| - \frac{1}{bx} + C$$

含有$\sqrt{a+bx}$的积分

$$\int x\sqrt{a+bx}\mathrm{d}x = \frac{2}{15b^2}(3bx-2a)(a+bx)^{\frac{3}{2}} + C$$

$$\int x^2\sqrt{a+bx}\mathrm{d}x = \frac{2}{105b^3}(15b^2x^2-12abx+8a^2)(a+bx)^{\frac{3}{2}} + C$$

$$\int x^n\sqrt{a+bx}\mathrm{d}x = \frac{2}{b(2n+3)}x^n(a+bx)^{\frac{3}{2}} - \frac{2na}{b(2n+3)}\int x^{n-1}\sqrt{a+bx}\mathrm{d}x$$

$$\int \frac{\sqrt{a+bx}}{x}\mathrm{d}x = 2\sqrt{a+bx} + a\int \frac{1}{x\sqrt{a+bx}}\mathrm{d}x$$

$$\int \frac{\sqrt{a+bx}}{x^n}\mathrm{d}x = \frac{-1}{a(n-1)}\frac{(a+bx)^{\frac{3}{2}}}{x^{n-1}} - \frac{(2n-5)b}{2a(n-1)}\int \frac{\sqrt{a+bx}}{x^{n-1}}\mathrm{d}x, n\neq 1$$

$$\int \frac{1}{x\sqrt{a+bx}}\mathrm{d}x = \frac{1}{\sqrt{a}}\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right) + C, a>0$$

$$=\frac{2}{\sqrt{-a}}\arctan\sqrt{\frac{a+bx}{-a}}+C, a<0$$

$$\int\frac{1}{x^n\sqrt{a+bx}}\mathrm{d}x=\frac{-1}{a(n-1)}\frac{\sqrt{a+bx}}{x^{n-1}}-\frac{(2n-3)b}{2a(n-1)}\int\frac{1}{x^{n-1}}\sqrt{a+bx}\mathrm{d}x, n\neq 1$$

含有$x^2\pm\alpha^2$的积分

$$\int\frac{1}{x^2+\alpha^2}\mathrm{d}x=\frac{\arctan\frac{x}{\alpha}}{\alpha}+C$$

$$\int\frac{1}{\pm x^2\mp\alpha^2}\mathrm{d}x=\frac{\ln\left(\frac{x\mp\alpha}{\pm x+\alpha}\right)}{2\alpha}+C$$

含有 ax^2+b的积分

$$\int\frac{1}{ax^2+b}\mathrm{d}x=\frac{1}{\sqrt{ab}}\arctan\frac{\sqrt{a}x}{\sqrt{b}}+C$$

含有 $ax^2+bx+c \qquad (a>0)$的积分

$$\int ax^2+bx+c\mathrm{d}x=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

含有 $\sqrt{a^2+x^2} \qquad (a>0)$的积分

$$\int\sqrt{a^2+x^2}\mathrm{d}x=\frac{1}{2}x\sqrt{a^2+x^2}+\frac{1}{2}a^2\ln\left(x+\sqrt{a^2+x^2}\right)+C$$

$$\int x^2\sqrt{a^2+x^2}\mathrm{d}x=\frac{1}{8}x(a^2+2x^2)\sqrt{a^2+x^2}-\frac{1}{8}a^4\ln\left(x+\sqrt{a^2+x^2}\right)+C$$

$$\int\frac{\sqrt{a^2+x^2}}{x}\mathrm{d}x=\sqrt{a^2+x^2}-a\ln\left(\frac{a+\sqrt{a^2+x^2}}{x}\right)+C$$

$$\int\frac{\sqrt{a^2+x^2}}{x^2}\mathrm{d}x=\ln\left(x+\sqrt{a^2+x^2}\right)-\frac{\sqrt{a^2+x^2}}{x}+C$$

$$\int\frac{1}{\sqrt{a^2+x^2}}\mathrm{d}x=\ln\left(x+\sqrt{a^2+x^2}\right)+C$$

$$\int\frac{x^2}{\sqrt{a^2+x^2}}\mathrm{d}x=\frac{1}{2}x\sqrt{a^2+x^2}-\frac{1}{2}a^2\ln\left(\sqrt{a^2+x^2}+x\right)+C$$

$$\int\frac{1}{x\sqrt{a^2+x^2}}\mathrm{d}x=\frac{1}{a}\ln\left(\frac{x}{a+\sqrt{a^2+x^2}}\right)+C$$

$$\int\frac{1}{x^2\sqrt{a^2+x^2}}\mathrm{d}x=-\frac{\sqrt{a^2+x^2}}{a^2x}+C$$

含有$\sqrt{x^2-a^2} \qquad (x^2>a^2)$的积分

$$\int \frac{1}{\sqrt{x^2-a^2}}\mathrm{d}x = \ln\left(x+\sqrt{x^2-a^2}\right)+C$$

含有$\sqrt{a^2-x^2} \qquad (a^2>x^2)$的积分

$$\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \arcsin\frac{x}{a}+C = -\arccos\frac{x}{a}+C$$

$$\int \sqrt{a^2-x^2}\mathrm{d}x = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}+C$$

$$\int x^2\sqrt{a^2-x^2}\mathrm{d}x = \frac{1}{8}x(2x^2-a^2)\sqrt{a^2-x^2}+\frac{1}{8}a^4\arcsin\frac{x}{a}+C$$

$$\int \frac{\sqrt{a^2-x^2}}{x}\mathrm{d}x = \sqrt{a^2-x^2}-a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)+C$$

$$\int \frac{\sqrt{a^2-x^2}}{x^2}\mathrm{d}x = -\frac{\sqrt{a^2-x^2}}{x}-\arcsin\frac{x}{a}+C$$

$$\int \frac{1}{x\sqrt{a^2-x^2}}\mathrm{d}x = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)+C$$

$$\int \frac{x^2}{\sqrt{a^2-x^2}}\mathrm{d}x = -\frac{1}{2}x\sqrt{a^2-x^2}+\frac{1}{2}a^2\arcsin\frac{x}{a}+C$$

$$\int \frac{1}{x^2\sqrt{a^2-x^2}}\mathrm{d}x = -\frac{\sqrt{a^2-x^2}}{a^2x}+C$$

含有$R=\sqrt{|a|x^2+bx+c} \qquad (a\neq 0)$的积分

$$\int \frac{\mathrm{d}x}{R} = \frac{1}{\sqrt{a}}\ln\left(2\sqrt{a}R+2ax+b\right) \qquad (\text{for } a>0)$$

$$\int \frac{\mathrm{d}x}{R} = \frac{1}{\sqrt{a}}\ \operatorname{arsinh}\frac{2ax+b}{\sqrt{4ac-b^2}} \qquad (\text{for } a>0,\ 4ac-b^2>0)$$

$$\int \frac{\mathrm{d}x}{R} = \frac{1}{\sqrt{a}}\ln|2ax+b| \quad (\text{for } a>0,\ 4ac-b^2=0)$$

$$\int \frac{\mathrm{d}x}{R} = -\frac{1}{\sqrt{-a}}\arcsin\frac{2ax+b}{\sqrt{b^2-4ac}} \qquad (\text{for } a<0,\ 4ac-b^2<0,\ (2ax+b)<\sqrt{b^2-4ac})$$

$$\int \frac{\mathrm{d}x}{R^3} = \frac{4ax+2b}{(4ac-b^2)R}$$

$$\int \frac{\mathrm{d}x}{R^5} = \frac{4ax+2b}{3(4ac-b^2)R}\left(\frac{1}{R^2}+\frac{8a}{4ac-b^2}\right)$$

$$\int \frac{\mathrm{d}x}{R^{2n+1}} = \frac{2}{(2n-1)(4ac-b^2)}\left[\frac{2ax+b}{R^{2n-1}}+4a(n-1)\int\frac{\mathrm{d}x}{R^{2n-1}}\right]$$

$$\int \frac{x}{R}\,\mathrm{d}x = \frac{R}{a} - \frac{b}{2a}\int \frac{\mathrm{d}x}{R}$$

$$\int \frac{x}{R^3}\,\mathrm{d}x = -\frac{2bx+4c}{(4ac-b^2)R}$$

$$\int \frac{x}{R^{2n+1}}\,\mathrm{d}x = -\frac{1}{(2n-1)aR^{2n-1}} - \frac{b}{2a}\int \frac{\mathrm{d}x}{R^{2n+1}}$$

$$\int \frac{\mathrm{d}x}{xR} = -\frac{1}{\sqrt{c}}\ln\left(\frac{2\sqrt{c}R+bx+2c}{x}\right)$$

$$\int \frac{\mathrm{d}x}{xR} = -\frac{1}{\sqrt{c}}\operatorname{arsinh}\left(\frac{bx+2c}{|x|\sqrt{4ac-b^2}}\right)$$

含有三角函数的积分

$$\int \cos x \mathrm{d}x = \sin x + C$$
$$\int \sin x \mathrm{d}x = -\cos x + C$$
$$\int \sec^2 x \mathrm{d}x = \tan x + C$$
$$\int \csc^2 x \mathrm{d}x = -\cot x + C$$
$$\int \sec x \tan x \mathrm{d}x = \sec x + C$$
$$\int \csc x \cot x \mathrm{d}x = -\csc x + C$$

$$\int \tan x \mathrm{d}x = -\ln|\cos x| + C = \ln|\sec x| + C$$
$$\int \cot x \mathrm{d}x = \ln|\sin x| + C$$
$$\int \sec x \mathrm{d}x = \ln|\sec x + \tan x| + C$$
$$\int \csc x \mathrm{d}x = -\ln|\csc x + \cot x| + C = \ln\left|\frac{\tan x - \sin x}{\sin x \tan x}\right| + C$$

$$\int \sin^n x \mathrm{d}x = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x \mathrm{d}x + C \quad \forall n \geq 2$$
$$\int \sin^2 x \mathrm{d}x = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\int \cos^n x \mathrm{d}x = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x \mathrm{d}x + C \quad \forall n \geq 2$$
$$\int \cos^2 x \mathrm{d}x = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\int \tan^n x \mathrm{d}x = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x \mathrm{d}x + C \quad \forall n \geq 2$$

$$\int \tan^2 x \mathrm{d}x = \tan x - x + C$$

$$\int \cot^n x \mathrm{d}x = \frac{1}{n-1}\cot^{n-1} x - \int \cot^{n-2} x \mathrm{d}x + C \quad \forall n \geq 2$$

$$\int \cot^2 x \mathrm{d}x = -\cot x - x + C$$

$$\int \sec^n x \mathrm{d}x = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x \mathrm{d}x + C \quad \forall n \geq 2$$

$$\int \csc^n x \mathrm{d}x = -\frac{1}{n-1}\csc^{n-2} x \cot x + \frac{n-2}{n-1}\int \csc^{n-2} x \mathrm{d}x + C \quad \forall n \geq 2$$

含有反三角函数的积分

$$\int \arcsin x \mathrm{d}x = x \arcsin x + \sqrt{1-x^2} + C$$

$$\int \arccos x \mathrm{d}x = x \arccos x - \sqrt{1-x^2} + C$$

$$\int \arctan x \mathrm{d}x = x \arctan x - \ln\sqrt{1+x^2} + C$$

$$\int \operatorname{arccot} x \mathrm{d}x = x \operatorname{arccot} x + \ln\sqrt{1+x^2} + C$$

$$\int \operatorname{arcsec} x \mathrm{d}x = x \operatorname{arcsec} x - \operatorname{sgn}(x)\ln\left|x+\sqrt{x^2-1}\right| + C = x \operatorname{arcsec} x + \operatorname{sgn}(x)\ln\left|x-\sqrt{x^2-1}\right| + C$$

$$\int \operatorname{arccsc} x \mathrm{d}x = x \operatorname{arccsc} x + \operatorname{sgn}(x)\ln\left|x+\sqrt{x^2-1}\right| + C = x \operatorname{arccsc} x - \operatorname{sgn}(x)\ln\left|x-\sqrt{x^2-1}\right| + C$$

含有指数函数的积分

$$\int e^x \mathrm{d}x = e^x + C$$

$$\int \alpha^x \mathrm{d}x = \frac{\alpha^x}{\ln \alpha} + C$$

$$\int x e^{ax} \mathrm{d}x = \frac{1}{a^2}(ax-1)e^{ax} + C$$

$$\int x^n e^{ax} \mathrm{d}x = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1} e^{ax} \mathrm{d}x$$

$$\int e^{ax} \sin bx \mathrm{d}x = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx) + C$$

$$\int e^{ax} \cos bx \mathrm{d}x = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx) + C$$

含有对数函数的积分

$$\int \ln x \mathrm{d}x = x\ln x - x + C$$

$$\int \log_\alpha x \mathrm{d}x = \frac{1}{\ln\alpha}\left(x\ln x - x\right) + C$$

$$\int x^n \ln x \mathrm{d}x = \frac{x^{n+1}}{(n+1)^2}\left[(n+1)\ln x - 1\right] + C$$

$$\int \frac{1}{x\ln x}\mathrm{d}x = \ln\left(\ln x\right) + C$$

含有双曲函数的积分

$$\int \sinh x \mathrm{d}x = \cosh x + C$$

$$\int \cosh x \mathrm{d}x = \sinh x + C$$

$$\int \tanh x \mathrm{d}x = \ln(\cosh x) + C$$

$$\int \coth x \mathrm{d}x = \ln(\sinh x) + C$$

$$\int \mathrm{sech}\, x \mathrm{d}x = \arcsin(\tanh x) + C = \arctan(\sinh x) + C$$

$$\int \mathrm{csch}\, x \mathrm{d}x = \ln\left(\tanh\frac{x}{2}\right) + C$$

定积分

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}\mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^{\frac{\pi}{2}} \sin^n x \mathrm{d}x = \int_0^{\frac{\pi}{2}} \cos^n x \mathrm{d}x = \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\dots\cdot\frac{4}{5}\cdot\frac{2}{3}, & \text{if } n>1 \text{ and } n \text{ is odd} \\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\dots\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}, & \text{if } n>0 \text{ and } n \text{ is even} \end{cases}$$